**Yield stress fluid flows: a review of experimental data**

P. Coussot
Laboratoire Navier, Univ. Paris-Est, Champs-sur-Marne, France

**Abstract**: Yield stress fluids are encountered in a wide range of applications: toothpastes, cement, mortar, foams, muds, mayonnaise, etc. The fundamental character of these fluids is that they are able to flow (i.e. deform indefinitely) only if they are submitted to a stress larger than a critical value, otherwise they deform in a finite way like solids. The flow characteristics of such materials are difficult to predict as they involve permanent or transient solid and liquid regions whose location cannot generally be determined a priori. Here we review the existing knowledge as it appears from experimental data, concerning flows of simple (non-thixotropic) yield stress fluids under various conditions mainly: (i) uniform flows in straight channels or rheometrical geometries, (ii) complex stationary flows in channels of varying cross-section such as extrusion, expansion, flow through porous medium, (iii) transient flows such as flows around obstacles, spreading, spin-coating, squeeze flow, elongation. The effects of surface tension, confinement, and secondary flows are also reviewed. A special emphasis will be laid on the experimental works providing information on the internal flow characteristics which give critical elements for a comparison with numerical predictions. It is in particular shown that (i) the deformations in the solid regime can play a critical role in transient flows; (ii) the yielding character does not play a significant role on the flow field when the boundary conditions impose large deformations; (iii) in secondary flows the yielding character is lost.

## 1. Introduction

A toothpaste flows like a liquid through the squeezed tube and spreads over the toothbrush where it will remain at rest during the displacement of the brush up to the mouth; then the agitation and friction against the teeth will widely deform it again. A mortar prepared in a container in which it is strongly mixed is then spread over a vertical wall to form a layer of a couple centimeters which will remain at rest from minutes to hours until it starts to settle (chemical reactions leading to solidification of the structure). A mayonnaise prepared by strong mixing will then be dispersed over the surface of some food and remain at rest there.

With these different fluids we are apparently dealing with materials able to keep the shape they have been given under gravity like solids and able to flow like liquids, i.e. being widely deformed, when submitted to a sufficiently high stress. A critical point is that if chemical reactions do not have time to play a role the transition between these two states is perfectly reversible: once at rest the material will soon recover the properties it had before flowing in the liquid regime. We are thus dealing with fluids in the sense that these materials can undergo any type of deformation without losing their intrisic mechanical properties.

According to the above description it is natural to distinguish two main states, or regimes, namely a solid regime and a liquid regime. There has been some brainstorming about the “true” existence of a solid regime, some authors [1] suggesting that it would be in fact a very high viscosity regime. Actually there is a clear strong transition of rheological behavior which justifies this distinction for most of the processes involving such materials [2]. Let us consider a typical commercial hair gel, it can flow easily like a simple viscous liquid if mixed with a spoon but the bubbles formed inside will remain apparently in the same place for years, as in a solid. Since for almost all applications we look at the mechanical properties of such materials over hours they will appear as solids if the stress applied to it is not too large.

Thus the fundamental character of these fluids is that they are able to flow (i.e. deform indefinitely) only if they are submitted to a stress larger than a critical value, otherwise they deform in a finite way like solids. The flow characteristics of such materials are difficult to predict as they involve permanent or transient solid and liquid regions whose location cannot generally be determined a priori. The situation is even more complex when one deals with yield stress fluids with properties depend on flow history, i.e. thixotropic fluids. Here we review the existing knowledge concerning flows of simple (non-thixotropic) yield stress fluids (see Section 2) under various conditions: (i) uniform flows in straight channels or rheometrical geometries (Section 3), (ii) complex stationary flows in channels of varying cross-section such as extrusion, expansion, flow through porous medium (Section 4), (iii) transient flows such as flows around obstacles, spreading, spin-coating, squeeze flow, elongation (Section 5). We also review the effects of surface tension, confinement, and secondary flows. In order to properly describe transient flows it appears critical to take into account the deformations in the solid regime.

## 2. Rheological characterization and context of the review

A rheometrical test makes it possible to better distinguish and characterize these solid and liquid regimes. It consists in a series of creep tests, i.e. the deformation is followed in time for a constant applied stress, for different stress levels (using a new sample of the same material for each value). After a transient stage the different deformation vs time curves evolve according to two ways (see Figure 1): for a stress larger than a critical value (say 62 Pa) they finally follow a slope 1 in logarithmic scale, indicating that the material flows at a constant shear rate, so that we will consider that the material is in its **liquid regime**; for a smaller stress the deformation seems to tend to a plateau, and remains below a critical value, as for a solid. It can still be argued that in the latter regime the deformation goes on increasing in time, suggesting a very slow flow. However the slope decrease in time indicates that the apparent shear rate would continuously decrease in time towards lower and lower values without reaching a steady state flow. The apparently limited deformation and continuous decrease of apparent shear rate towards very low values justifies that we consider this regime as a **solid regime**. It must nevertheless be mentioned that for stresses close to the critical value the situation is not so clear: the deformation seems to be able to reach significant value but the shear rate significantly decreases (slope less than one in logarithmic scale). This illustrates the difficulty to determine precisely the yield stress value: ideally the yield stress is the stress associated with a steady flow at an infinitely small shear rate which, due to the finite deformation associated with the solid-liquid transition, would need an infinite time to be reached.

Actually such a behavior description, i.e. with two different mechanical regimes, is in fact difficult to accept with regards to usual concepts in material physics. A solid behavior is generally associated with a given structure, and a solid material breaks or flows depending on whether it is respectively brittle or ductile, but anyway it loses its initial mechanical properties beyond some critical deformation associated with the breakage of the initial structure. The amazing aspect of yield stress fluids is that: (i) they exhibit a solid structure at rest, which may be considered in the same frame as usual solids; (ii) beyond the critical deformation this structure breaks and the material flows as a simple fluid; (iii) if the material is not left again at rest it recovers it initial mechanical (solid) properties associated with a similar structure. Thus we are dealing with a material able to move from a solid to a liquid state, and from a liquid to a solid state, in a reversible way. Such a situation likely results from the soft interactions between the elements composing the fluid, which can form a jammed structure with an apparent solid behavior, but can also easily breaks if submitted to sufficiently large stress.

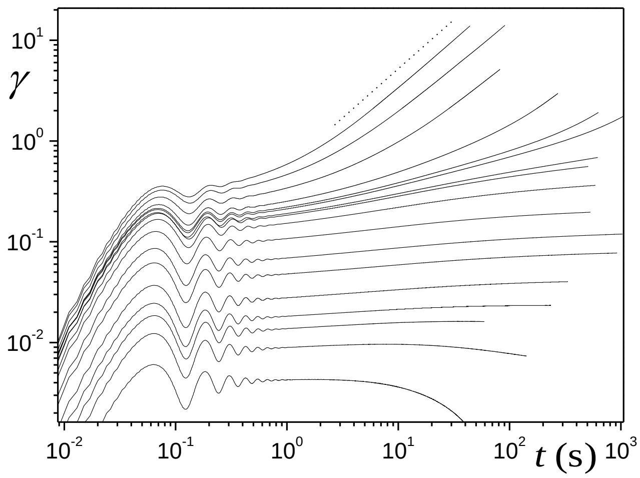


**Figure 1**: Typical aspect of deformation vs time curves for different stress levels (from bottom to top) applied to a hair gel: 0.8, 4, 6, 8, 12, 20, 28, 40, 50, 56, 60, 63.2, 66.8, 70, 80, 90, 100 Pa. The inclined dotted line is the curve of slope 1. Data from [3]

Such a description seems to very well apply to materials such foams, gels, emulsions, but not as simply for colloidal suspensions whose structure can significantly evolve at rest as a result of colloidal interactions and Brownian agitation. In that case the transition between the solid and the liquid regimes as described above seems to be more "abrupt". In a series of creep tests a jump appears between the two regions, which finally means that no steady state can be reached at a shear rate value below a critical one [4-5]. Such a behavior was described as intimately associated with the thixotropic character of the fluid [6]. A first consequence of such a behavior is the development of a shear-banding when the material is submitted to an apparent shear rate below this critical value [7-10]. Another consequence is that one cannot so easily define the yield stress of the material: starting from rest we may identify a *static yield stress*, i.e. the critical stress allowing steady state flow, which nevertheless can significantly vary with the time the material has been left at rest before (since the structure strengthened in time); decreasing progressively the stress applied when the material is flowing, one may identify a *dynamic yield stress*, i.e. the final stress applied before complete stoppage; however this value can depend on the exact flow history, and in particular the time spent at high shear rates [4].

Here we will mainly focus on the flow characteristics of non-thixotropic yield stress fluids which can as a good first approximation be described as *simple yield stress fluids* [11-12], i.e. with identical static and dynamic yield stress whatever flow history. This will also provide some basic elements for the description of flows of weakly thixotropic yield stress fluids for which the flow history is rapidly forgotten during flow. However, in some cases we will comment on the possible impact of strong thixotropic effects on some of the flow characteristics.

A wide set of data in the last twenty years has shown that for shear rates in the range $\left[0.01;100\mathrm{s}^{-1}\right]$ the rheological behavior of simple yield stress fluids in steady state simple shear can be very well represented by the Herschel-Bulkley model, which writes as follows:

$\tau<\tau_c \Rightarrow \dot{\gamma}=0$ (solid regime); $\tau>\tau_c \Rightarrow \tau=\tau_c+k\dot{\gamma}^n$ (liquid regime) (1)

in which $\tau$ is the shear stress amplitude, $\dot{\gamma}$ the shear rate amplitude, $\tau_c$ the yield stress and $k$ and $n$ two material parameters. A tensorial 3D expression is extrapolated from (1) with the help of a von Mises criterion for the solid-liquid transition [13]. The parameter $n$ is generally found in the range [0.3; 0.5]. In practice the yield stress value is generally in the range [1-100 Pa]. Indeed, measurements outside this range are difficult due to various experimental problems (surface tension effects, apparatus sensibility, sample fracture for high yield stress, etc).

By definition this simple model does not encompass any aspect of the transient behavior of the material and in particular its possible elastic and/or viscous properties in the solid regime. This aspect has in fact not been studied in depth so far, people mainly focused on the behavior in the liquid regime. It seems that the rheological properties of yield stress fluids in the solid regime vary as widely as for usual solids, ranging from almost purely linear elasticity (for concentrated emulsions) [14-15] to complex elastoplasticity for cement paste [16] through non-linear viscoelasticity for Carbopol gels [3]. A common aspect is nevertheless that starting from rest the material must deformed up to a critical deformation before reaching the liquid regime.

In the following we will only consider laminar flows for which inertia effects are negligible. This can be considered to be the case if the generalized Reynolds number, i.e. $\mathrm{Re}=\rho V^2/\tau_c$ in which $\rho$ is the fluid density and $V$ a characteristic velocity, is smaller than 1, but obviously as for turbulence in simple fluids the laminar regime may extend up to much larger Reynolds number depending on the initial and boundary conditions. In the context of laminar flows of yield stress fluids a dimensionless number appears of fundamental interest for characterizing the flows. This is the Bingham number, $\mathrm{Bi}=\tau_c/k\dot{\gamma}^n$, which estimates the ratio of the constant (related to the strength of the solid structure) to the rate-dependent (viscous) parts of the constitutive equation, and finally gives an idea of the relative importance of the solid and liquid regions in the sample.

There has been a lot of numerical simulations of yield stress fluid flows in a wide range of applications (e.g. [12, 17-30]), which generally rely on simplified assumptions for the description of the rheological behavior, although more sophisticated approaches are now appearing [31-40]. The basic approach consists to assume that below some shear rate the fluid is not solid but highly viscous [41]. When a true solid regime is taken into account [42] the possible deformations in the solid regime are nevertheless generally neglected. It is likely that for some ranges of parameters these works provide relevant results. However these predictions are far from fully validated due to a strong lack of experimental data, and there also exist various experimental results which do not seem to be well predicted by such simulations. In the present review we intend to review the main existing experimental works providing an insight in the flow properties of yield stress fluids, and we will mention numerical works only when they are helpful for our understanding of the process. We will more particularly focus on the experimental works providing information on the internal flow characteristics which provide critical elements for a comparison with numerical predictions.

We will distinguish three types of flows: uniform flows (Section 2), non-uniform steady flows (Section 3), and transient flows (Section 4). In the two first situations the solid and liquid regions are established but not in the last case. This makes it possible to review a wide set of simple and complex flows such as capillary flows, flows through porous medium, displacement of a solid object, coating, spreading over a solid surface, elongational flows, squeeze flows, etc. We will not discuss here more complex flows which so far have been more occasionally the object of experimental studies: this is in particular the case for thermal instabilities [43-44], boiling [45], bubble dynamics [46-47], mixing [48], impact of droplet on solid surface [49-52], impact of a solid projectile on a fluid bath [53], jet flow perpendicular to a solid surface [54], two-phase flows [55].

Note that with yield stress fluids a slip at the wall may have a much stronger impact on the flow characteristics than for any other fluid type. Indeed if wall slip occurs a yield stress fluid can flow steadily (possibly as a plug) even if submitted to stresses much smaller than the yield stress, which is a critical behavior change. The best way to avoid any problem of that type is to use rough wall surfaces, with a roughness significantly larger than the maximum size of the components of the fluid. Here we will not consider the impact of a possible wall slip on the different flow types, assuming that the no-slip condition is valid in any case.

## 3. Uniform flows

Here we define a uniform flow as one for which there is a direction along which the flow characteristics are identical in any plane perpendicular to this direction. This direction can be a straight line or a circle line. Uniform flows are thus obtained through long straight conduits of any (constant) cross-section and in rotational geometries such as those used in rheometry (parallel disks, coaxial cylinders, cone and plate). Free surface flows in channels of fixed shape can also be uniform if the wetted perimeter remains constant.

### 3.1 Flows in cylindrical geometries

*Cone and plate geometry*

Flows in cone and plate geometry with small cone angle is an ideal situation since the stress distribution can be considered as homogeneous. Thus the flow (a simple shear) is a priori homogeneous too, either in the solid or in the liquid regime. Under these conditions a single deformation or a single shear rate is associated to a given shear stress. In fact a variation of a few percent of the shear stress throughout the gap can exist as a result of the conical sample shape and/or the imperfect shape of the sample periphery. For a yield stress fluid this may have a significant impact on the shear rate distribution at low shear rates. For example, for a (slow) flow at $\mathrm{Bi}=20$, with a maximum relative stress variation ($\Delta\tau/\tau$) of 5% throughout the gap, some region can be arrested ($\tau<\tau_c$) while the rest of the material is flowing ($\tau>\tau_c$). This implies that the effective shear rate in the flowing region differs from the apparent shear rate, i.e. the ratio of the rotation velocity to the cone angle. The extent of the arrested region, and thus the discrepancy between the apparent and the effective shear rate, grows when the apparent shear rate decreases (i.e. when $\mathrm{Bi}$ increases). This means that even with this geometry we can hardly expect to get relevant data concerning the effective behavior of the material from macroscopic measurements at the approach of the yield stress.

Observation of the flow characteristics inside such a geometry are rare. The basic, and, as far as we know, the unique experiment with simple yield stress fluids, was carried out by Magnin and Piau [56] in a pioneering work, by looking at the deformations of a vertical mark drawn at the periphery of the sample (a Carbopol gel) before starting to rotate the cone. A homogeneous deformation was observed at sufficiently large apparent shear rates. At apparent shear rates below about $0.01 s^{-1}$ some kind of internal slippage apparently develops in the gel. This effect differs from the progressive spatial variation of shear rate expected as a result of stress variation in the sample (see above). It seems that here this slippage occurs at the end of the solid regime, when the material starts to yield but is unable to develop a homogenous flow in the liquid regime.

Other observations of the flow of yield stress fluids in cone and plate geometry concern thixotropic materials (Bentonite suspension [10] and Ludox [9]). In that case it was shown that a shear-banding effect develops at apparent shear rate ( $\dot{\gamma}$ ) below a critical value ( $\dot{\gamma}_c$ ) depending on the material: the material is sheared at $\dot{\gamma}_c$ in a thickness $h = H\dot{\gamma}/\dot{\gamma}_c$ (where $H$ is the gap) and remains in a solid state in the rest of the gap. This effect again differs from those described above since now there is both a flow in a layer of significant thickness and an abrupt transition between the solid and the liquid region.

*Parallel disks*

In parallel disks the apparent shear rate is strongly not homogeneous in the radial direction. At a rotation velocity $\Omega$ the apparent shear rate at a distance $r$ expresses as $\Omega r/H$ , which means that it varies linearly from 0 along the central axis to $\Omega R/H$ . As a consequence it is extremely difficult to predict the exact stress field. Anyway the deformation undergone by the central region is much smaller than the material at the periphery, it also varies linearly from 0 to a maximum value $\theta R/H$ if $\theta$ is the angle of rotation. Thus, if we start from rest we will have regions at the periphery reaching the liquid regime much before those along the central axis. There is a technique for analyzing the data of parallel disks which takes into account the shear rate heterogeneity and determine the effective constitutive equation in simple shear (cf. [13]). The above considerations show that for yield stress fluids this technique is strictly valid only when the liquid regime has been reached everywhere in the sample, otherwise, in particular for start up flows, two different rheological behavior types must be taken into account in two regions and the analysis is much more difficult.

*Couette flows*

Couette flow constitutes a very interesting case since the stress distribution is heterogeneous along a radial direction but it is known a priori from the momentum equation. The shear stress writes $\tau = M/2\pi r^2$ , where $M$ is the torque applied per unit depth. For a purely elastic behavior in the solid regime any stress value gives rise to some deformation, in proportion to this stress. As a consequence we expect a deformation in this regime similar to that obtained for a Newtonian fluid after a given time of flow, namely the material is deformed in the whole gap whatever the gap size and the deformation slowly decreases with the distance from the inner cylinder. On the contrary, in the liquid regime there may exist an unsheared region beyond a radius $r_c$ such that $r_c = \sqrt{M/2\pi\tau_c}$ . Although this might provide critical information on the effective behavior of such materials there does not exist much observation of the solid-liquid transition under those conditions. This is likely because velocity measurements generally need significant time of acquisition and significant displacement over this time, which implies that a steady flow is much preferable. Indirect measurements of such deformation in the solid regime in time were carried out from a mathematical analysis of the macroscopic deformation resulting from a series of start-up flows at different stress levels [57].

Apparently, since the deformation is the largest along the inner cylinder, the solid-liquid transition occurs first in this region, and rapidly progresses until reaching its maximum value (see Figure 2).

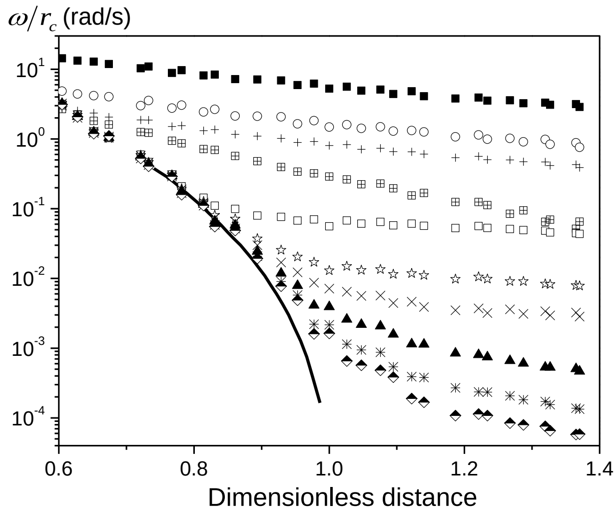


**Figure 2**: Profiles of rotation velocity (scaled by $r_c$) as a function of the dimensionless distance $r/r_c$ in the gap of a Couette system during a creep test (at a stress level larger than the yield stress) at increasing times (from top to bottom): 0.005, 0.015, 0.03, 0.1, 0.5, 1, 2, 5, 15, 32 s. The continuous curve corresponds to the steady state velocity profile. In the first times the material is in its solid regime and deforms everywhere, which leads to a significant velocity everywhere. Then a region close to the inner cylinder starts to flow in the liquid regime and progressively increases in size. During this period the motion in the solid region is reduced and its apparent velocity significantly decreases down to zero. Data from [58]

The steady state velocity profiles in a Couette geometry have been measured by MRI for a series of materials (gels [59], Emulsions [60], Foams [61]) and indeed correspond to those expected for a material following a Herschel-Bulkley behaviour (1). In those cases a consistent behavior (see e.g. Figure 3), deduced from local stress and shear rate measurements under different flow conditions (at different rotation velocities of the inner cylinder) was obtained and was shown to be in agreement with data obtained from conventional rheometrical tests with a cone and plate [59]. However it was also shown that for shear rates below about $0.01\mathrm{s}^{-1}$ it did not appear possible to get relevant data from MRI and from conventional rheometry, likely because the flow is unstable and leads to some localization. This situation in fact corresponds to the creep flows observed at stresses (56 and 60 Pa) very close to the yield stress (see Figure 1) which can neither be associated with a flow stoppage in the solid regime nor with a steady flow at constant shear rate since the apparent shear rate continuously decreases in time.

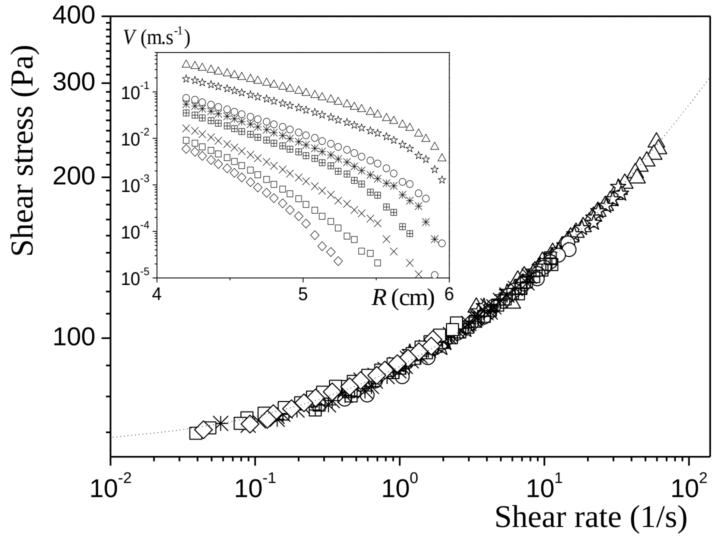


**Figure 3**: "Local" constitutive equation of a Carbopol gel deduced from stress and velocity distribution measured by MRI in steady state flows in a Couette geometry (inner cylinder 8 cm diameter, outer cylinder 10 cm diameter). The dashed line corresponds to a Herschel-Bulkley model fitted to data. The inset shows, with the same symbols, the corresponding velocity profiles at the different rotation velocities (from bottom to top): 2, 3, 5, 10, 15, 20, 50, 100 rpm. Data from [59].

A wide set of data concerning the internal flow characteristics of Carbopol gels in small gap Couette cells with smooth walls was obtained in recent years by the Manneville's group using ultrasonic velocimetry coupled with conventional rheometry in a wide range of rotation velocities. These results showed that the transient behavior of those yield stress fluids might be quite complex: beyond the critical deformation a shear-banding regime develops whose duration decreases as a power-law of the applied shear rate; then the thickness of the shear-band evolves and the whole gap is sheared (see Figure 4) [62]. These characteristics, which were not observed in geometries with larger gap such as those shown in Figure 3, were correlated to the existence of a slight stress overshoot in the (increasing) flow curve. The impact of some parameters such as wall roughness, gap size, etc was then studied in depth [63-64] but this effect remains unexplained.

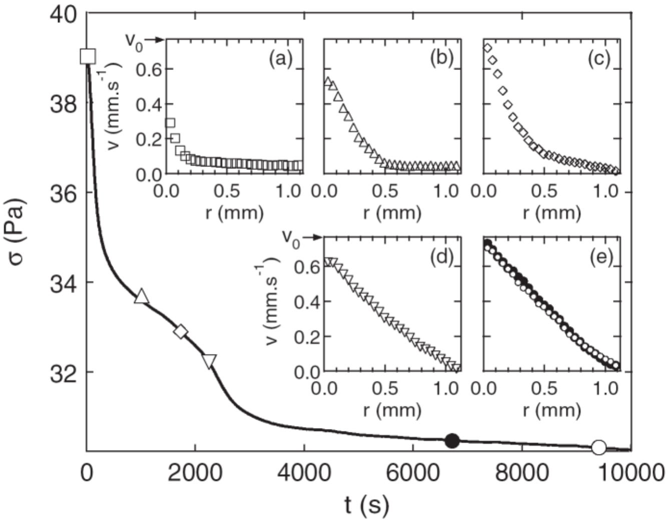


**Figure 4** : Flow of a Carbopol gel in a Couette geometry with a small gap and rough walls. Shear stress vs time for a shear rate of $0.7\,s^{-1}$ applied at the initial time. The insets show the corresponding velocity profiles at different times: (a) 27 s, (b) 1013 s, (c) 1730 s, (d) 2250 s, (e) 6800 s (filled circles) and 9400 s (open circles). Data from [62].

For some years a technique appeared, i.e. the LAOS (Large Amplitude Oscillations Shear) [65], which intends to take advantage of the principle of oscillating flows, which proved to be so useful for viscoelastic liquids, for approaching the complex behavior of yield stress fluids. Here the large amplitude makes it possible to get out of the solid regime, at least during some part of the motion. However during such a process the material undergoes a complex flow history made of undetermined periods in the solid regime or in the liquid regime, which significantly complicates the interpretation of data, even if recent works have provided keys for a qualitative and sometimes quantitative interpretation of such results [66-68]. An advantage of this technique seems to be that it studies in a single test the various aspects of the rheological behavior. One disadvantage is that it becomes more difficult to interpret the data in a straightforward way and we easily miss direct observations of some possible additional complications such as those above mentioned, i.e. transient shear-banding, localization at low shear rates, wall slip. The interpretation of data for thixotropic yield stress fluids is obviously even more tricky. In order to measure in a straightforward way the basic rheological parameters of yield stress fluids it is preferable to use simpler procedures such as: (i) a series of creep tests as presented above from which one can deduce all the steady state properties in the solid and liquid regimes, (ii) a sweep test to estimate the flow curve in the liquid regime, (iii) a low velocity shear to estimate the yield stress (from the stress peak), all procedures which allow to directly follow the state of the material in time.

3.2 Flow in conduits or channels

There are other situations in which the stress distribution is known: flow between two parallel planes, through a cylinder, or free surface flow above a wide plane.

*Flow between two parallel planes*

This flow is interesting as it provides the basis for some theoretical considerations but it is not often encountered in practice. We consider the flow though a channel of rectangular cross-section, and note $y$ the distance from the mid-plane. The channel width is sufficiently large compared to its thickness so that we can assume that edge effects are negligible. In that case we generally assume that the pressure ( $p$ ) is uniform in each cross-section and only varies along the flow direction. From the momentum equation we deduce that we have a simple shear flow with a distribution of the shear stress in the form $\tau = y\nabla p$. This implies that the fluid is unsheared in a central region defined by $|y| < -\tau_c/\nabla p$, so that this region, i.e. the so-called "plug", remains in the solid regime and moves at the maximum velocity. This plug flow is often used as the typical characteristics of yield stress fluid flows even if, as we will see below, it effectively develops only in uniform flows.

*Flow through a capillary*

Following the same approach as above we deduce that a central plug exists in the region $r < 2\tau_c/\nabla p$. Although the stress is again not homogeneous in the fluid as for a Couette flow (see above) it is possible to derive the constitutive equation from a series of tests at different flow rates (see [13]). However such flows have not been the object of many studies. One reason is that they are difficult to carry out in a wide range of flow rates: indeed it is difficult to slowly control and vary the pressure drop imposed. In practice a bath of fluid is generally used, over which a pressure is applied, but it is then difficult to control precisely the start-up flow around some pressure drop because a slight variation of the level of fluid in the bath can induce a slight variation of the applied pressure. On the other hand we can attempt to impose the flow rate but the problem now is the measure of the pressure drop. There is no clear means to get a relevant measure of the pressure with yield stress fluid: any type of measurement can be affected by the residual (normal or tangential) stresses that may exist in the fluid at rest and thus in its solid regime. Another possible critical problem with capillary flows of yield stress fluids is that wall slip can easily occur and it is more difficult to roughen the inner surface of a capillary than of other geometries. As for a Couette flow (see [13]), assuming that wall slip depends only on the shear stress, it is possible to deduce the value of wall slip from a series of tests at different flow rates in capillaries of different sizes. However this is a long and complex procedure.

Observations of the local properties of capillary flows of yield stress fluids are rather scarce. As far as we know this started with the experiments of Corvisier [69] using PIV and ultrasound with laponite suspensions. The fluid was mixed before entering the conduit. The velocity profile appear to be Newtonian at the entrance then progressively develops a central plug region likely as a result of a progressive restructuring of the fluid leading to an increase of its yield stress. A further analysis is nevertheless extremely difficult since we are here dealing with a thixotropic fluid for which a single flow curve does not exist (it depends on the exact flow history). Other experiments were carried out with starch suspensions in Silicone oil [70], which showed the basic trends expected for a yield stress fluid but no systematic analysis or comparison with independent rheometrical measurements confirmed the consistency of the observed behavior. A successful comparison between rheometry and velocity field in a capillary in the absence of wall slip was done by Rabideau [71] for a Carbopol gel but only at one flow rate and with a rather small sheared region. Recently a more systematic study of the flow development associated with pressure variations in time by increasing then decreasing steps was carried out [72]. The authors in particular showed that (i) a hysteresis between the increasing and decreasing pressure ramp curves occurs, similarly to that observed in sweep tests in rheometry with rotational geometries and likely due to elastic effects during the solid-liquid transition; and (ii) a progressive transition towards steady state flow with a velocity profile consistent with theory by taking into account wall slip (see Figure 5).

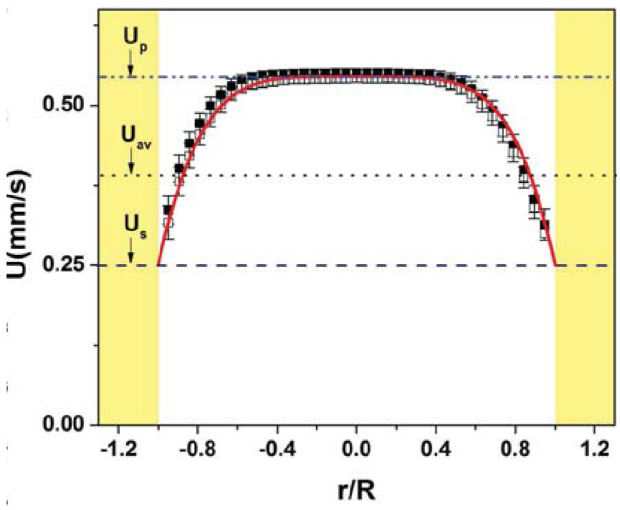


**Figure 5** : Steady state flow through a pipe : longitudinal velocity as a function of the dimensionless distance from the central axis. The continuous line corresponds to the theoretical velocity profile taking into account a slip velocity $U_s$ . The dotted line shows the level of the average velocity. Data from [72].

For the flow through a conduit of more complex cross-section the stress distribution is a priori unknown but we again expect to have a plug region, possibly small, at a sufficient distance from the wall since the shear stress tends to zero towards some central axis. Flow through ring-shaped conduits, i.e. flow between two coaxial or eccentric cylinders along the direction of their axis, have been the object of many theoretical studies (e.g. [73-74]) as they play a critical role in drilling operations. However we are not aware of experimental studies attempting to study in a systematic way the validity of models or looking at the local flow properties.

*Gravity flow over an inclined plane*

The theoretical characteristics of the gravity flow over an inclined plane are similar to those in a half plane of a planar Poiseuille flow (see above). This is the same for the flow along a filled semi-cylindrical open channel: the flow characteristics are similar to those in a half part of a cylindrical conduit.

The flow over a wide inclined plane has been studied with mud suspensions at different concentrations, varying the channel slope ( $i$ ) and flow discharge [75]. A general consistency of the data with the theoretical predictions using the rheological behavior of the material determined from independent measurements was found in that case. It is remarkable that the critical thickness associated with stoppage of a wide uniform layer of a yield stress fluid is simply related to the yield stress through $h_c = \tau_c / \rho g \sin i$ , in which $\rho$ is the fluid density and $g$ the gravity. Some approximate treatment of the wall effects for smaller aspect ratios was also suggested, assuming that it acts as an additional stress with a shear rate equal to the mean value along the plane. This provides a practical technique for measuring the yield stress with an inclined channel plane [76-77]. Additional macroscopic data of that type exist in the form of a data base but do not seem to have been so far analyzed in depth [78].

More precise studies were recently carried out [79] using a recirculating flume which makes it possible to have a long duration, steady uniform flow far from the edges. The rheological properties of the materials (Carbopol gels, Kaolin suspensions), the flow rate and channel slope were varied and the flow thickness was measured. Careful independent rheometrical tests were made so that a full comparison of the theoretical predictions with experimental data could be done. Surprisingly an excellent agreement between the detailed flow characteristics was obtained for kaolin suspensions, but unexpectedly not for Carbopol gels, as if the effective yield stress was larger by a factor 1.1 to 1.2. This discrepancy remains to be explained. Moreover the velocity profiles inside the fluid were measured using PIV (see Figure 6). The flow characteristics deduced from theory, in particular the plug region, could thus be observed directly, and a good agreement with the predictions using the Hesrchel-Bulkley model was found taking into account the correction factor above mentioned for the gels.

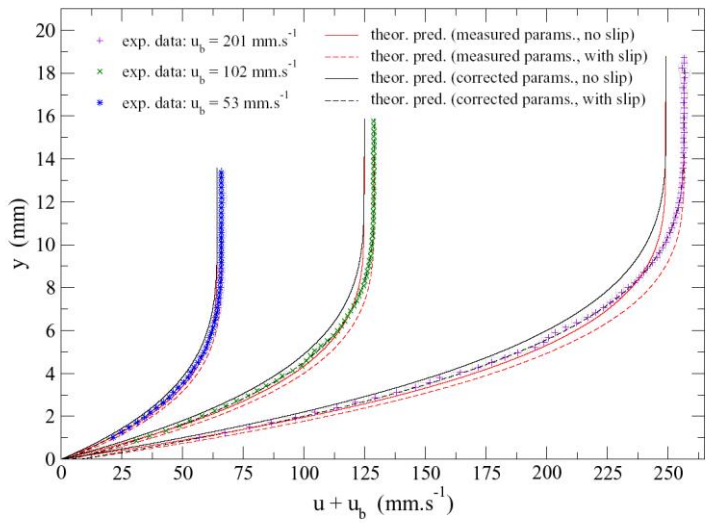


**Figure 6** : Steady state uniform flow of a Carbopol gel over a moving inclined plane (at a velocity $u_b$ ) : profiles of the velocity (with regards to the plane surface) as a function of the height above plane for different plane velocities. The different theoretical curves correspond to the usual theory for a Herschel-Bulkly fluid taking into account or not a slight wall slip and/or a correction on rheological parameters (see text). Data from [79]

When the free surface flow of a fluid is too fast or too thin, i.e. for a sufficiently large Froude number [80], the uniform flow disappears and an instability occurs which takes the form of roll waves. A useful generalized approach of this phenomenon for non-Newtonian fluids was proposed by Trowbridge [81] and applied to yield stress fluids [75]. This type of approach was then refined by Balmforth and Liu [82] taking into account the internal specificities of yield stress fluid flows, i.e. including an almost undeformed region. In practice the specific trend of roll waves with such fluids is that between two successive rolls the flow may stop because the depth is too small [75]. This instability has been rarely studied experimentally with yield stress fluids. The attempt to test the validity of the basic theoretical criterion [81] from tests with mud suspensions in channels provided a reasonable agreement [75]. Another study used (thixotropic) bentonite suspensions modelled with a Bingham model [83], which is clearly not relevant and precludes a full consistent comparison between theory and data. Note that the characteristics of a single, stable wave of yield stress fluid was studied in depth with the help of a recirculating flume [54], providing a nice insight of the flow characteristics inside the material, in particular at the surge front where the fluid recirculates.

## 4. Stationary flows in non-uniform geometries

Here we consider steady-state flows with non-uniform boundary conditions, so that there cannot be a uniform flow. The steady-state character of these flows means that at any point the flow characteristics does not vary in time. The possible situations of that type are steady flows through more or less complex solid geometries, ranging from a slightly non-uniform conduit to a porous medium. Here we will essentially consider a few ideal geometries of that type.

### 4.1 Flow through a straight conduit of slowly varying cross-section

In the case of a slowly varying cross-section we can assume as a first approximation that the local flow characteristics are given by those for the steady flow through a uniform conduit of cross-section equal to the local one. As far as we know no experimental data, which could confirm this assumption, exist in that case. This situation was nevertheless studied, in their pioneering work, by Denn and Lipscomb [85] from a theoretical point of view, with the basic aim to discuss whether a plug region can exist in a non-uniform flow. The authors showed that, strictly speaking, a rigid plug cannot exist since as the conduit cross-section varies the plug size and/or shape is expected to vary, which is in contradiction with a rigid body assumption: a cross-section layer of this body would be continuously reshaped as it advances along the conduit.

In fact this conclusion should be tempered: in its solid regime a yield stress fluid may be somewhat deformed without yielding (up to about 50% for some gels). As a consequence, if the length of the conduit is not too large so that the total deformation, undergone by the fluid elements in the region supposed to correspond to the plug, is smaller than the critical one, they will remain in the solid regime. In that case a solid region (and not a “rigid” region), somewhat deformed, will persist along the conduit.

This situation could in fact be observed in the extreme case of the flow through a sudden expansion followed by a contraction (“a cylindrical box”) [86]: MRI measurements of the velocity field made it possible to estimate the deformation of fluid elements at different positions in a cross-section; it appeared that the fluid in the plug of the uniform upstream flow (before entering the box) remains mainly in its solid regime during its motion through the box. Finally, at low velocities the flow through a box takes the spectacular (approximate) form of a plug going through a wide volume of yield stress fluid at rest.

### 4.2 Flow in a rapidly expanding conduit

Here we consider the entrance flow through a long conduit of much larger size. We expect that the uniform flow in the downstream conduit establishes only after some length. Between this region and the entrance section the size of the flowing region increases until reaching the whole conduit, while there exist dead zones in the corner regions around the entrance flow. In contrast with what may be observed with simple or viscoelastic liquids the fluid in the dead zones is effectively arrested and thus in its solid regime. After some length along the downward conduit a new uniform flow is established. We are not aware of experimental observations of that situation so far.

### 4.3 Flow in a rapidly reducing conduit section, i.e. extrusion

There has been a large push to understand and control the key phenomena surrounding the forming and shaping of polymer materials by extrusion [87]. Much less well known is the extrusion behavior of yield stress materials such as toothpastes, foodstuffs, ceramic slips, plasters, mortars, etc, although this method is often employed with such materials to achieve the products final form. Once formed, the product can be dried, fired or treated in some other manner to achieve its final form.  In that field most works were focused at determining mathematically [88] or numerically (e.g. [89-90]) the entrance correction for the flow of a Bingham fluid through an orifice, i.e. the additional pressure drop strictly due to the change of section. Efforts to verify the above formulae experimentally have mainly been concerned with pastes having a very high yield stress [91], for which it was difficult to determine the rheological parameters separately.

Direct observations inside the materials with clay paste [92], biscuit dough and soap [93], and Carbopol gels [71], showed that a plug flow exist at a some distance above and below the die entrance, and the fluid is static in the corners around the entrance. Thus there is a converging zone of acceleration of the fluid just before the die (see Figure 7). Another effect is the poor dependence of the shape of the flow field on the overall flow rate for Bingham number larger than 1, i.e. the velocity field scaled by the maximum velocity did not vary significantly (see Figure 7) [71, 94]. Surprisingly a similar result, i.e. similar flow fields after rescaling, was obtained for thixotropic materials after significantly different times of rest [95]. At last the flow field in the large section seems to be essentially independent of the contraction ratio (see Figure 8), i.e. the ratio of the two section radii.

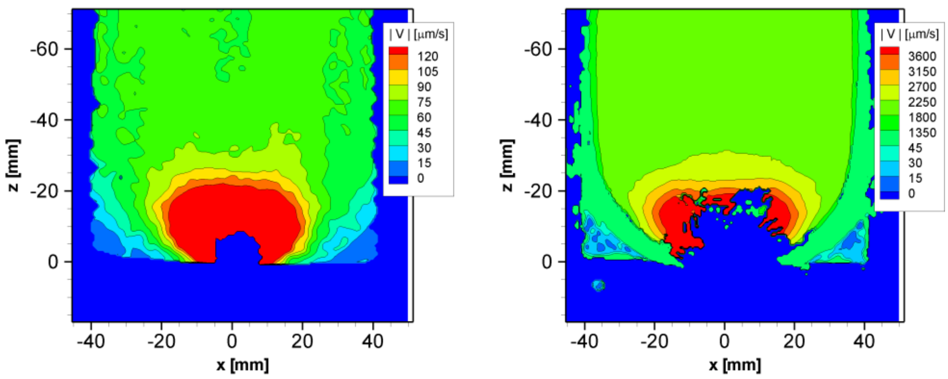


**Figure 7**: Steady state flow field in the upper zone of an 8:1 contraction with imposed piston velocities (in the large conduit) of (left) 60 μm/s and (right) 1800 μm/s. Data from [75]

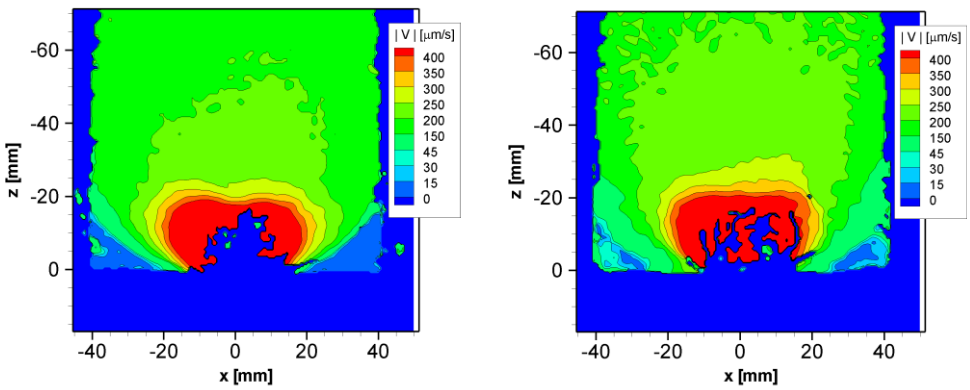


**Figure 8**: Steady state flow field in the upper zone with imposed piston velocities (in the large conduit) of (left) 200 μm/s for (left) a 40:1 contraction and (right) a 4:1 contraction. Data from [75]

The results of numerical simulations did not appear in excellent agreement with these data [75] but a better agreement was found with data concerning kaolin pastes [94]. The main difference between the two types of materials likely concerns the critical deformation: the Carbopol gel remains in its solid regime in a wider range of deformations. This implies that in the entrance region where the plug region is deformed elastic effects might have a significant impact on the flow properties.

Similar experiments were carried out in a somewhat different frame [96]: 2D flow of a foam through a contraction; several characteristics appeared similar to those of the 3D flow above (acceleration around the entrance, slight variations of the flow characteristics with the flow rate). In contrast with usual rheological studies such an experiment gives access to a rich set of information concerning the local stresses. However the extrapolation to real 3D situations with more finely divided systems is still questionable in particular because in these tests slip occurred along the side walls and the bubble diameter was not much smaller that the die diameter.

## 4.4 Flow through a box of finite length

The flow through a box of finite length corresponds to a sudden enlargement followed by a sudden reduction. A set of data was obtained with Carbopol gels through boxes by photos with long exposure times [97] making it possible to follow the trajectories of micron-sized spheres dispersed in the fluid. Thus the solid and the liquid regions could be clearly distinguished since the former yield limited displacements whereas the latter yield trajectories extending over the full image if the exposure time is sufficiently long. It was shown that the fluid flows through a central channel, while the surrounding fluid is at rest (see Figure 9). As expected from qualitative considerations about the stress distribution this channel tends to expanse in the middle of the box for higher Bingham number and longer boxes. A further study using MRI velocimetry [98] provided information on the velocity profile for Bi>1 and a box of length equal to twice the conduit diameter. In that case the flow channel diameter is close to that of the conduit, a plug zone region persists in this channel, and the shear rate in the transitional region is almost constant as for a shear-banding effect [98]. The original aspect of such a flow is that the stress at the two sides of this transitional region must be equal to the yield stress, so that we finally have a flow of a plug lubricated by a liquid layer but a stress at the outer wall of this layer equal to the yield stress.

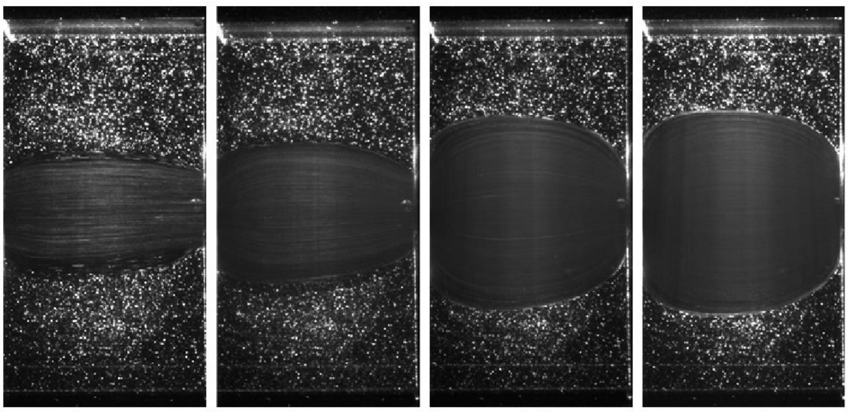

**Figure 9** : Flow of a Carbopol gel through a box (expansion ratio 5:1) at different flow rates expressed in terms of the wall stress (in small conduit) to yield stress ratio (from left to right): 3, 5.3, 8.4, 10.5. The dark central zones correspond to significant motion (flow) of the suspended particles over the experiment duration, whereas the particles (light points) appear as fixed in the arrested (solid) regions surrounding the central zone. Data from [97].

## 4.5 Flow through a porous medium

Flows of yield stress fluids through porous media is of interest for various applications: the injection of slurries or cement grouts to reinforce soils, penetration of glue in porous substrates and, likely the most economically important application, injection of drilling fluids in rocks either for the reinforcement of the wells or for enhancing oil recovery. For Newtonian fluids (of constant viscosity $\mu$) we know that any element of the liquid network flows and in the absence of inertia effects the velocity field scaled by the average velocity ($\overline{V}$) is constant (Stokes flow). This is at the origin of Darcy's law which tells us that the pressure gradient ($\nabla p$) is proportional to $\overline{V}$ by a factor $\mu/K$ in which $K$ is a characteristic of the porous medium, namely its permeability. The existing approaches for establishing the pressure drop vs flow rate relation for yield stress fluids have relied on the usual physical approaches of the permeability coefficient appearing in the Darcy's law for Newtonian fluids: they use the analogy of the flow at a local scale within the porous medium with the flow either through a capillary or around isolated particles. Thus one can obtain an equation relating the pressure drop to the average velocity involving specific but partly unknown lengthscales (pore size, total flow path length). In this way various expressions for $\Delta p = f(V)$ have been obtained [99-104] which all have the following structure, typical of yield stress fluids: the velocity differs from zero only beyond a critical pressure drop $\Delta p_c$; then it progressively increases as the difference between the actual pressure drop and $\Delta p_c$ increases. Besides there have been some approaches based on network models [105] which provide more straightforward descriptions of the reality but don't provide generic analytical expressions for the pressure drop vs velocity relationship.

Some in-depth physical studies suggested that two critical effects could occur: (i) at the pore scale the flowing volume increases with the pressure gradient [106] and (ii) at a macroscopic scale the flow starts as a percolation effect, i.e. at a critical pressure drop liquid regions exist only along a specific path throughout the porous medium [107] and as $\nabla p$ is increased more flowing paths progressively form within the porous medium. Such effects still need an experimental validation. In fact these effects should not lead to an apparent behavior at a macroscopic scale fundamentally different from that which has been observed or predicted: they are consistent with a critical pressure threshold and then an increase of pressure drop with velocity. The point is that the lengthscales involved in this law might not vary at all as expected in the above theoretical approaches.

A few experimental studies exist in that field [99-101] but they often rely on parameter variations (pore size, rheological parameters, mean velocity) in rather limited ranges. Actually a major difficulty in experiments of non-Newtonian fluid flow through porous media is the same as for flow through conduits (see above), namely to be able to control and/or measure properly the pressure drop, but in addition it is critical to check that there is no particular interaction between the fluid and the porous media (impurities modifying the fluid properties, jamming of the elements in the pores, etc). Existing data converge towards an expression for the pressure drop vs velocity formally similar to the Herschel-Bulkley expression:

$$\nabla P = \nabla P_c + k_p V^n \qquad (2)$$

Such an expression is in agreement with the above theoretical considerations above. In a more systematic recent experimental study it was shown [108] that the parameters scale with the rheological parameters and bead size (for a bead packing) as: $\nabla P_c = \alpha \tau_c / D$ and $k_p = \beta k / D^{n+1}$, where $\alpha$ and $\beta$ are two parameters which should be independent of the material type and porous medium characteristics but in fact different values were found for $\alpha$ for emulsions and Carbopol gels.

Anyway this expression does not give any information concerning the possible existence of arrested regions and their variation with flow rate, and more generally we have no information on the physical origin of the two parameters in (2). Usual imaging techniques can hardly yield detailed information at the pore scale. Some interesting information may nevertheless be obtained from NMR measurements of the probability density function of the velocity with the help of pulse-gradient field technique [109-110] which provides straightforward data on the average local flow characteristics without being affected by any spatial resolution problem (the signal from all proton spins is recorded). With the help of this technique it was shown [111] that, in contrast with the expected result, for Bingham number in the range 1 to 12 the region of fluid at rest is negligible even at low $\overline{V}$ values. Moreover the velocity density distribution appears to be similar to that of a Newtonian fluid. A plausible explanation is that in a rapidly varying geometry such as a porous medium the deformation is imposed to the fluid so that the role played by its constitutive equation on the flow characteristics is minor (although its role on the stress is still major). These results finally made it possible to deduce the effective form of the Darcy's law for yield stress fluids and the physical origin of its coefficients which appear to be related to the distribution of the second invariant of the strain rate tensor in the fluid volume:

$$\alpha = (1/\varepsilon\Omega)\int 2\frac{D}{V} d_{II}\,\mathrm{d}\omega \;;\; \beta_n = (1/\varepsilon\Omega)\int\left(2\frac{D}{V} d_{II}\right)^{n+1}\mathrm{d}\omega$$

## 5. Transient flows

Here we consider flows which are not steady, i.e. for which there exists at least a region of the fluid in which the flow characteristics significantly evolve in time. Various such flows exist and it is likely that we will not be quite exhaustive in this review.

### 5.1 Spreading flows

The most common transient flow is the spreading of a finite volume of yield stress fluid over a plane. It is the simplest flow for which one can observe in practice the yielding character of the fluid since, instead of spreading widely more or less rapidly (depending on its viscosity) like a simple liquid, a yield stress fluid stops flowing after some time and forms a deposit of significant thickness. Actually this thickness is proportional to the yield stress value and when it is too small surface tension effects may play a role on the stoppage conditions, as for simple liquids.

Such a flow may be induced by different ways. The most common one consists to pour a given volume of material over a given point of the solid surface. The material then forms, over the solid surface, a heap which grows and spreads at the same time, and finally stops flowing after a certain time since the end of pouring. The shape of the deposit formed under such conditions was first studied theoretically by Coussot [112]. As a first approximation, for an inclined plane, as long as the fluid thickness is smaller than the critical one associated to stoppage of a uniform layer on the same slope (i.e. $h_c$, see Section 3.2), the fluid keeps the shape it has been given during pouring (see Figure 10 left). If the fluid volume is sufficiently large so that $h_c$ was overcome during the process, the liquid regime has been reached and the equilibrium corresponds to a shear stress equal to the yield stress, so that the final shape can be predicted from momentum balance [112]. In that case the thickness far from the edges is equal to $h_c$ (see Figure 10 right).

**Figure 10**: (left) Deposit of a foam sample on a shaving brush. Since the volume is small and the yield stress is large the fluid keeps the shape it was given during set up. (right) Deposit of a mud layer over an inclined plane after flow stoppage. The material was initially stored behind the gate upstream. After opening the gate it spreads in an almost uniform layer with a thickness around 2 cm.

The shape and growth of deposits formed over inclined slopes with other boundary conditions was studied in the aim of approaching the flow characteristics of natural events such as mudflows or lava flows: spreading over a conical surface [113], extruded inclined dome [114], extrusion from a source on an inclined plane [115], spreading from an upward source [116]. In the latter case it was demonstrated [116] that the fluid moves along the steepest slope direction but at the same time continuously spreads in the transversal direction, it does not form a channel of given transversal extent as can be assumed from a simple approach [117]. This additional extent results from the pressure gradient associated with the transversal variation of thickness at the edge, even if a similar pressure gradient along the steepest slope would not induce a flow by itself. The transversal flow here occurs because as long as there is a longitudinal flow (along the steepest slope) the yielding criterion is overcome and the material behaves as a simple fluid without yield stress in any other direction. This effect is similar to that described below for example when coupling a squeeze and a shear flow.

These results explain the properties of deposits of yield stress fluids formed over inclined surfaces, and provide excellent practical means for measuring the yield stress of a material without any sophisticated apparatus. However so far these predictions have been only roughly confirmed by a few measurements [112, 115] but discrepancies were also noticed when a well-controlled material was used [118-119].

There might still be some aspect not well taken into account in the above theories, but more likely the discrepancies between theory and experiments are due to the difficulty to have flow conditions in agreement with those assumed in the related theory. Indeed in general the theories assume that the fluid deposit is formed at the end of a flow in its liquid regime since this is the only situation for which the stress is known in the solid regime obtained at stoppage. But such a situation is rather difficult to control in practice during the processes of injection, extrusion, coating or pouring, which are used to spread the material over the slope.

For a horizontal plane the deposit shape is different: the thickness does not reach a critical value in the central part of the deposit but instead the central thickness tends to infinity when the sample volume is increased; the theory predicts a parabolic shape for the deposit with a central thickness increasing with the horizontal extent [112]. As for the deposit over inclined planes there has been confirmation of the validity of this technique for some materials [120] but there does not exist a systematic experimental verification of the validity of this theory and it was even suggested that in some cases normal stresses could play a significant role [121]. This approach has been extended to the flow stoppage in a channel, i.e. the L-box in civil engineering or Botswick consistometer in food industry [122-123].

The so-called “slump test”, widely used in civil engineering for estimating the handling ability of a cement-based material involves such a flow. However here, in order to control the flow the fluid is initially set up in a truncated cone or cylinder which is suddenly removed so that the material then spreads more or less widely over the horizontal solid surface. When the fluid widely spreads the above theory likely applies since the deposit is formed at the end of a well-developed flow in the liquid regime. On the contrary, when it only slightly flows there remains an almost undeformed region at the top of the deposit (see Figure 11). It is likely that this region has remained in the solid regime. Simple theoretical approaches were suggested for describing the flow characteristics of the yielded region (below the solid part) assuming it is submitted to a simple elongational flow [120, 124] the extent of which varies with the weight of material situated above. A more sophisticated approach was also developed recently [125]. This makes it possible to obtain a formula giving the “slump” (i.e. the height difference between the initial and final states of the deposit) as a function of the yield stress.

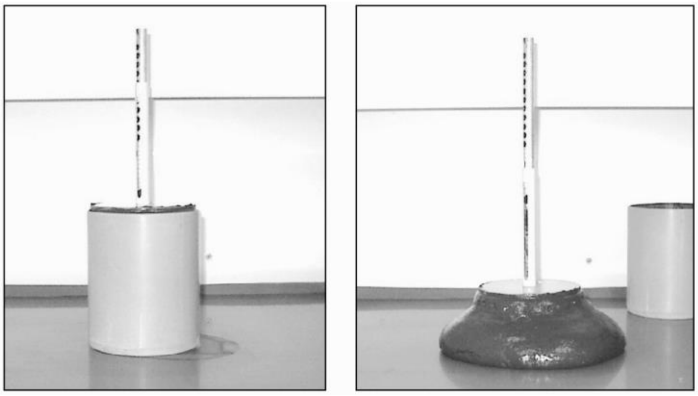

**Figure 11**: Slump of a sewage mud (right) after withdrawing the cylinder initially keeping it in this shape (left). Photo from Baudez [126].

Another way to supply the material over the inclined plane consists to open the gate behind which it has been stored over the inclined plane. The flows of viscoplastic fluids resulting from such dam-break has been studied theoretically in recent years: either specifically the initial flow and the final spreading before stoppage [127-128], or the whole flow characteristics from initiation [129]. Comparisons with experimental results were more or less successful [128, 130-131] but once again the transition from the initial solid regime to the liquid regime is not easy to control in the tests and to take into account in the theory. Finally the specific impact of the viscoplastic character of the fluid on the flow characteristics under such conditions are still unclear, as suggested by the good agreement of the predictions of a simple kinematic wave model [132] with some tests.

5.2 Spin coating

In a spin-coating process the fluid is put on a solid surface which is then rapidly rotated so that the fluid spreads radially. Here the fluid is submitted to a horizontal acceleration which plays a role qualitatively analogous to that of gravity for the flow over an inclined plane. The spin-coating of yield stress fluids has been the subject of a few theoretical studies. The authors [133-135] essentially addressed the difference from Newtonian films, in particular demonstrating that the fluid thickness cannot be uniform. This applies to the final fluid thickness at stoppage and simply results from the increase of the volume force (centrifugal force) with the radial distance. Experiments [136] showed that a sample of material starts to flows beyond a critical rotation velocity. As the rotation velocity increases successive equilibrium profiles are then obtained, which take a complex shape at high velocities, i.e. the sample separates into two parts, one remaining at rest around the rotation axis, the other going on moving radially in the form of a circular roll (see Figure 12) [136]. Besides it was shown that small droplets of yield stress fluids tend to deform in the form of fingers [137].

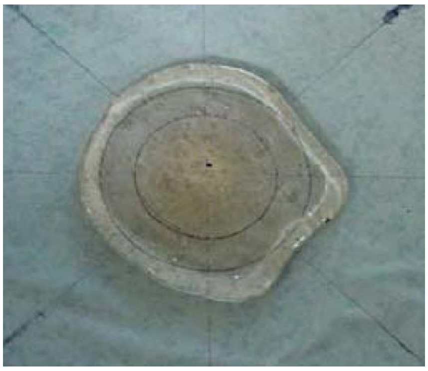

**Figure 12** : Shape of a deposit of yield stress fluid (initially cylindrical and lying at the central point) after a rapid rotation of the plate (spin coating). Photo: H. Tabuteau [136].

5.3 Squeeze flows

Here we consider the flow in a layer situated between two solid plates which are progressively approached to each other. If the layer thickness is much smaller than its extent we can use the lubrication assumption which shows that we have a dominant simple shear flow imposed by a pressure gradient. It is then possible [13, 138] to compute the velocity field and deduce, from the integration of the pressure, analytical expressions for the normal force on the plate as a function of the mean velocity in the large and small asymptotic limits (however the intermediate regions remain to be solved numerically). Adams et al. [139] also suggested a closed form solution over the entire range of velocities with limited error in the intermediate regions. These theories in particular lead to a simple theoretical expression for the critical thickness at stoppage for a given normal force ($F_0$) and a given sample volume ($\Omega_0$) as a function of the yield stress of the material:

$$b_c = \left( \frac{\tau_c^2 \Omega_0{}^3}{18\pi F_0{}^2} \right)^{1/5} \tag{3}$$

Note that these approaches are only approximations of the reality since they predict the existence of a central plug, which is not possible for the reasons expressed before (see Section 4.1) [85]. Another, more critical aspect is that one assumes that before stoppage the whole fluid was in the liquid regime. That means that the deformation undergone since the beginning of the test is everywhere larger than $\gamma_c$. However, more or less as for the shear rate in a parallel disk geometry (see Section

3.2) this deformation strongly varies from the center of the sample, where it is of the order of $\Delta b/b$, to its periphery where it is of the order of $(R/2b)\Delta b/b$.

The first studies examining the reliability of the squeeze flow test for determining a paste's yield stress considered an assortment of various common household soft materials [140-141]. A series of constant force steps were employed following the evolution of the inter-plate separation towards a limiting height thus determining the yield stress. In these studies, smooth glass plates were used as the contacting surface, a detail that could easily lead to imprecise measurement. Later studies acknowledged the importance of a rough surface for limiting slip effects [142-145]. The results of these experiments were compared with rheological measurements performed with both a four-bladed vane and serrated parallel plates. A global agreement was found which, in some instances, resulted in substantial discrepancy, such that there still remains a significant uncertainty. A systematic study performed on different concentrations of Carbopol with varying yield stresses carefully controlling the experimental conditions led to yield stress values that agree with the rheological measurements within 10% [146]. The same study also checked the validity of the lubricational theory for describing the force vs velocity in time. A further study [147] checked the no-slip assumption via direct observations of the flow field. It also focused on strong squeeze flow (at high velocities) and showed that in that case a strain-rate independent viscosity may be used to describe the macroscopic properties.

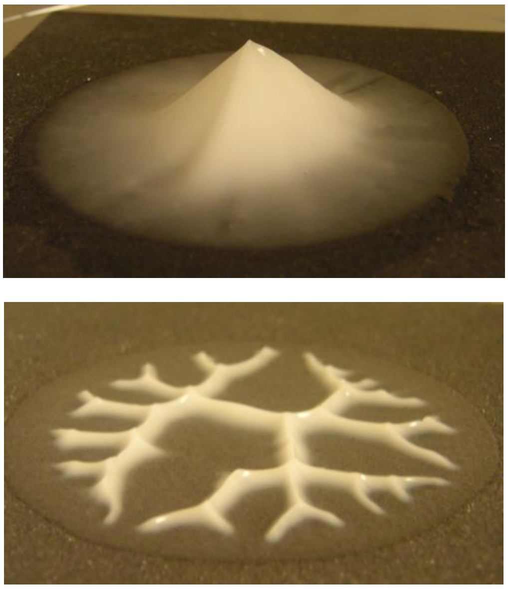

**Figure 13**: Main aspects of the sample deposit at the end of an adhesion test: (upper) large initial aspect ratio (thickness to radius ratio), main material volume in a central region surrounding by a thin film of fluid; (ii) small initial aspect ratio, uneven shape (fingering) of the main material volume. Photos Q. Barral [148]

The opposite process ("adhesion test"), i.e. obtained by moving away two plates initially separated by a layer of paste, should at first sight lead to the opposite flow characteristics, with the same relation between the force amplitude and the plate velocity. However it was shown that in general the force is larger than expected (up to two times the force for squeeze flow under the same geometrical conditions and at the same rate) [148]. The explanation suggested is a pinning effect related to the flow history: initially the material is in contact with a wide area of the solid plate and the line of contact remains fixed during the process so that there remains a thin layer of fluid behind the fingertip penetrating the bulk during plate withdrawal (see Figure 13); the stresses in this thin layer could induce an additional normal force [148]. This effect was also considered to stabilize the interface precluding viscous fingering in the range expected for the development of the Saffman-Taylor instability (see Section 5.5).

### 5.4 Elongational flows: instability and drops

Using a 3D expression for the stress tensor as a function of the strain rate tensor the elongational flow of an initially cylindrical volume of a yield stress fluid can be described theoretically as long as the sample shape remains cylindrical (see e.g. [149] or [120]). In particular one can get the expression for the critical normal stress that can be supported by the sample at rest. This stress is proportional to the yield stress but different results are obtained depending on the exact boundary conditions in the system and in particular the distribution of normal forces applied at the outer surface of the sample.

However, in practice everybody can observe that, except for a yield stress of the order of a few Pascals, a volume of yield stress fluid, whatever its initial shape, does not keep its cylindrical shape while it is elongated [149-151]: there is a pinch off process of the cylinder, most of the sample is slightly deformed while the central region rapidly gets thinner and finally the sample breaks into two conical parts (see Figure 14) [148, 150, 152-153]. Since this effect does not correspond at all to that expected from the basic theory we are here dealing with a kind of instability and the exact velocity field is unknown, especially around the separation region. This instability likely occurs because of the special nature of these fluids, and more precisely the possible coexistence of solid and liquid regions of arbitrary relative volumes.

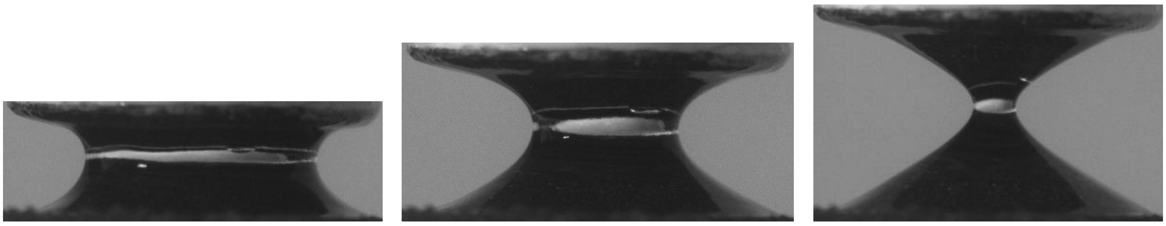

**Figure 14**: Evolution of the shape of a small drop (initially 2 mm diameter) of yield stress fluid ($\tau_c = 34\,\mathrm{Pa}$) in time as the upper plate is move upwards (from left to right). Photo J. Boujlel [153].

In order to describe the pinch off process it was suggested [149] that the flow can still be described as a simple elongational flow in each fluid disk even close to this region, which after some calculation makes it possible to predict the time elapsed before separation when the fluid is extruded at constant rate. However this prediction is not in excellent agreement with the experimental data. Balmforth et al. [151] provided a further insight in the shape of the drop during separation, which in particular explains the final conical shape of the deposits obtained in such tests.

Besides it was recently found [154] that the pinch off dynamics is independent of the fluid behaviour for Newtonian, shear-thinning and yield stress fluids. This result might be explained by the effect already mentioned in the case of flows through porous media (see Sections 4.5): when a yield stress fluid undergoes a homogeneous complex flow it loses its yielding character and tends to exhibit a flow field similar to those of a Newtonian fluid.

In literature there appears to remain some ambiguity on the effective value of the viscous force associated with the elongational flow of a yield stress fluid. Some works were specifically aimed at determining the critical force of separation from either a drop elongation using a Capillary Breakup Elongational Rheometer (CaBER) [152], or a drop formation and fall from a die exit [156]. In each case the measure relies on the critical force at breakage, which is supposed to be balanced by capillary effect. A uniaxial elongational flow is assumed in the fluid around the region of breakage. In that case the theory for a simple uniaxial elongation and using the usual 3D form for the Herschel-Bulkley constitutive equation of the form [13] predicts a normal force due to viscous effects (in the limit of large $Bi$) equal to $\sqrt{3}\tau_c$ times the cross section of the fluid cylinder. In the first case [152], with emulsions, the normal viscous force was found to be larger by a factor 2.8 whereas in the second case [155], with Carbopol gels, the correct theoretical factor (i.e. 1) was found. Recently a more complete study of the Laplace pressure in filaments [156] of various types of yield stress fluids led to the conclusion that the yield stress factor is equal to 1 only if the measurement is taken at a ratio of the filament life time to fluid relaxation time equal to 1. However another set of data [153] with small droplets of Carbopol gels showed that the normal viscous force all along the extension process at low rates is simply equal to the yield stress times a constant factor around 3.

It has been suggested [152] that the discrepancy between the theory and experiments concerning this problem would be due to the fact that the effective constitutive equation should also include a term depending on the third invariant of the stress tensor. Another explanation for this discrepancy was also suggested [153]: as the drop is elongated an increasing fraction of the sample, in contact with the upper and lower plates, stops flowing; this means that the flow progressively occurs only in a small central region the thickness and diameter of which continuously decrease; as a consequence it is not so clear that we have a simple uniaxial elongation in this region; there might also be some shear and we know that for a squeeze flow in the lubricational regime (small thickness ($b$) to diameter ratio) the normal stress due to flow is equal to $\tau_c R/3b$, which can be significantly larger than $\sqrt{3}\tau_c$ if the flow is restricted to a thin layer $b << R$. The situation is likely different for a drop formation due to the exit through the die as in [149, 155] since in that case an extension is the natural process induced by gravity. On the contrary for the drop breakage between two plates the contact of the fluid with the plates tends to fix the drop diameter at a short distance from the region of breakage.

## 5.5 Displacement of an object through a yield stress fluid

The displacement of solid objects through a yield stress fluid is a mechanism which needs to be understood in order to control sedimentation of coarse particles or bubbles in various material types (drilling fluids, fresh concrete, detergent, cosmetic, sauce, etc). Besides, in various processes involved in the preparation or use of pasty materials (mixing, cooking, eating) a tool is introduced and moved through the material initially at rest in a container. This has led to the set up of a practical test (penetrometer) for characterizing the mechanical strength of pastes which is used in a lot of standard tests for a variety of pasty materials such as skin creams, paints, edible fats, greases, mortars, etc. In this test a solid cylinder, cone, needle, or perforated disk, is pushed under a given force through the material until stoppage. The penetration depth is considered as a “rheological” characteristic of the material. Actually the resisting viscous force increases mainly because the surface of contact between the object and the fluid increases. Indeed, if we assume that the viscous stress along the solid surface remains of the order of the yield stress we understand that the total viscous force will increase more or less in proportion to the surface of contact.

The questions to be addressed concerning the displacement of an object through a yield stress fluid are basically the force vs velocity relation and the deformation field, i.e. what is the extent of the perturbation induced by the object motion, as a function of the object shape and the rheological characteristics of the material.

This problem has so far been mainly studied in the case of a compact object. The displacement of a sphere or a cylinder moving through a yield stress fluid has been studied in depth via theory and simulations assuming an undeformed rigid region [42, 157-161]. Force vs velocity expressions have been deduced and partly confirmed experimentally (although some unexplained discrepancies remain) [162-165]. Basically the force follows a Herschel-Bulkley type expression with an apparent shear rate equal to the ratio of the velocity to an "apparent" sheared thickness proportional to the sphere radius. The drag force on a cylinder [166] and on a disk [167] were also recently determined from systematic experiments with well-controlled yield stress fluids.

Only a few works aimed at measuring directly the flow field around the object. Atapattu et al. [162] were the first to measure the tangential velocity profiles around a sphere. Gueslin et al. [168] provided an in-depth study of the velocity field but this concerned a thixotropic material for which there can be a further, and maybe dramatic, impact of the intense shear rate close to the object surface which would tend to liquefy specifically this region. Finally Putz et al. [169] (for spheres) and Tokpavi et al. [166] (for cylinders) provided new detailed data concerning the velocity field around an object moving through a yield stress fluid. They found a significant discrepancy of the velocity field with that found from simulations. In particular they remarked an asymmetry on the flow field, the extent of the sheared zone ahead of the object being larger than expected from theory. A significant flow asymmetry was also observed in 2D foam flow around an obstacle [170] which was attributed to elastic effects.

It is likely that a critical issue in some of these works, which nevertheless might not explain the observed asymmetry, is the following: although they are generally considered as the flow characteristics of the fluid in the liquid regime, the measurements concern the apparent velocity field which could include both flow in the liquid regime and deformations in the solid regime. Actually this flow type poses a serious theoretical problem. In simple geometries (Couette, straight conduit, open-channel flow) for which the stress distribution is given, the solid-liquid interface is fixed after some time of flow and the distribution of the material in the liquid and solid regions does not change any more during the flow. For the displacement of an object, even a long plate, there is a change of paradigm: as the object advances, even in steady motion new solid regions are continuously reached and liquefied. This means that new deformations of the material in the solid regime continuously occur around the object and might play a significant role in the process.

This conclusion was confirmed by the analysis of the flow field around an object of simpler form, i.e. a plate, moving through a bath of yield stress fluid [171]: although the fluid seems to exhibit a complex velocity distribution from the instantaneous velocity field, it was shown that only a layer of small thickness along the plate effectively flows in the liquid regime while the rest of material undergoes small total deformations in the solid regime. Finally the deformations in the solid regime allow the particle to displace through it, i.e. it tends to decrease the amount of fluid liquefied. A critical example was observed with a thixotropic material [172] in which the liquid layer was extremely small and the bead displaced essentially by deforming the solid around it leaving behind a very thin liquefied filament.

In the case of a vertical plate displacement through a bath of yield stress fluid a curious phenomenon was observed: a uniform region in the liquid regime develops along the plate, and the thickness of this region varies very slightly with the plate velocity. For the opposite situation, i.e. a plate withdrawn from the bath of yield stress fluid, a similar effect occurs inside the fluid and the liquid region has a thickness close to that for plate penetration [173]. Note that the shear rate is the liquid layer is almost constant (see inset of Figure 15) as for the region of separation between the two solid regions for the flow through a box (see Section 4.1). It appears that the thickness of the layer coated on the plate increases linearly with the yield stress and is a fraction of the liquid layer thickness in the bath [174]: the fluid simply bifurcates at the approach of the free surface (see Figure 15).

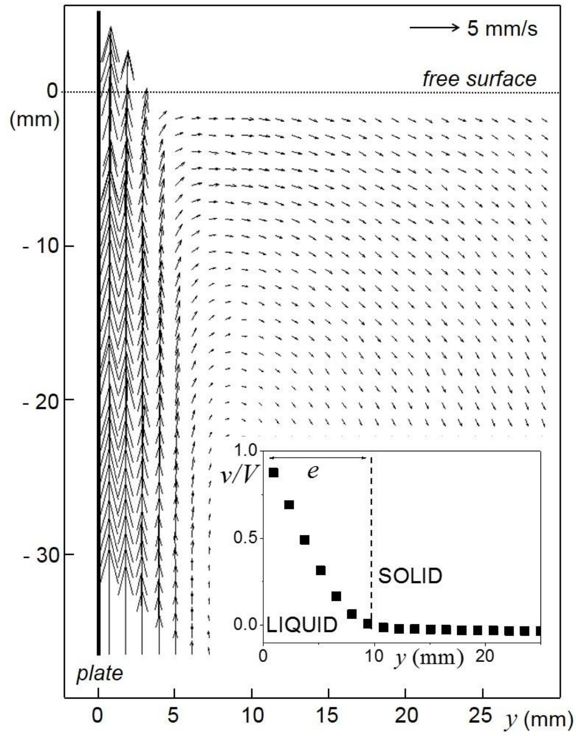


**Figure 15**: Dip coating of a yield stress fluid ( $\tau_c = 34\,\mathrm{Pa}$ ): velocity field along the plate inside the bath for *V*=15mm/s. The arrow length is proportional to the velocity modulus. The inset shows the steady uniform profile of the longitudinal velocity measured beyond 4 cm from the free surface. The vertical dotted line shows the position of the interface between the liquid layer and the solid region. Data from Maillard [174].

## 5.6 Other specific effects

*Secondary flows*

A particle of higher density suspended in a yield stress fluid in its solid regime can stay at rest if it is sufficiently small and the yield stress is sufficiently large. This simply follows from the fact that the drag force on an object moving through a yield stress fluid is equal to the sum of a constant term plus a term varying with the velocity. However, when this particle is suspended in the fluid while this fluid is already flowing in its liquid regime in some direction, the particle settles. This effect was observed by MRI [175]: dense particles at rest in the fluid at rest start to settle as soon as the fluid is (fully) sheared horizontally in the gap of a Couette geometry.

This is so because in its liquid regime the fluid has in some sense provisionally "forgotten" its yielding character. Indeed its constitutive equation is now only expressed as a straightforward equation relating the stress tensor to the strain rate tensor without any yield criterion, and the apparent viscosity depends on the flow rate but it is finite in any case. As a consequence, for example if one induces a secondary flow to the fluid already submitted to a simple shear ( $\dot{\gamma}$ ), i.e. a much slower flow than the first one, the apparent behavior of the fluid in this secondary flow will be that of a Newtonian fluid of apparent viscosity given by $\tau/\dot{\gamma}$ , which is finite for any value of $\dot{\gamma}$ , i.e. there is no yielding criterion leading to an infinite apparent viscosity. The validity of this analysis was shown in the case of sedimentation [175-176] and squeeze-shear flow [176-177] in which either the squeezing or the shearing was the main flow and the other one the secondary flow.

*Saffman-Taylor instability*

When two solid surfaces separated by a layer of yield stress fluid are moved away from each other the fluid flows so as to gather around a central point and form a layer of larger thickness (see Section 5.2). However, if the layer thickness is sufficiently small, or the yield stress or the separation velocity sufficiently large a fingering effect develops (see Figure 13), so that layers of material of significant thickness are left behind while the other fluid fraction still flows towards the central point [148]. Such an effect may be observed easily in our everyday life with puree, glues, mortars, etc, after some coating with a tool or the hand followed by the separation of the two solids. This phenomenon has in fact all the characteristics of the Saffman,-Taylor instability: it develops when the yield stress fluid is pushed by a less viscous one (here air), and when viscous effects in the pushed fluid are sufficiently large.

A theoretical approach of the Saffman-Taylor instability for yield stress fluids provided a criterion for the development of the instability in a straight channel or in a radial flow [178]. Further studies focused on the case of dominant viscous effects (i.e. low Bingham number) [179-180]. However the basic theory [178] was shown to be unable to predict the experimental data concerning the occurrence of the instability in both cases [148]. For a radial flow the instability occurs much later, for a critical value larger by several orders of magnitude than predicted by the theory [148]. It was suggested that this results from the deposits of material which stay behind the advancing front along the wall and tend to stabilize the front. This effect is indeed not taken into account in the theory which assumes, as for simple fluids, a perfect dewetting of the walls at the front. So there is still a lot to do to understand and characterize properly this phenomenon.

*Confinements effects*

In recent years, with the help of techniques of observation such as confocal microscopy, the flow characteristics of yield stress fluids were studied at the scale of the components of the material. Thus it was shown that a description of the flow within the frame of the continuum assumption becomes invalid when the characteristic length of the flow (typically the channel width) is smaller than several tenths the element size. More precisely, the constitutive equation determined from “macroscopic” tests (with a much larger characteristic length) is unable to describe these flow characteristics at small scale. This effect was observed from careful analysis of emulsion flows in microchannels (see Figure 16) [181], rheometrical tests on Carbopol gels with parallel disks [182], squeeze flows with Carbopol gels and commercials sauces [183]. This was interpreted as an effect of “confinement”. Actually it seems that this confinement effect is intrinsically linked to the yielding behavior. Indeed this yielding character implies that we have a jammed structure formed with mesoscopic elements negligibly submitted to thermal agitation. This structure needs to have a size much larger than its basic elements otherwise it will have characteristics depending on local arrangements. Finally a specific theoretical frame for the description of flow properties in such case was developed, introducing a new kind of constitutive equation based on a concept named “fluidity” [184].

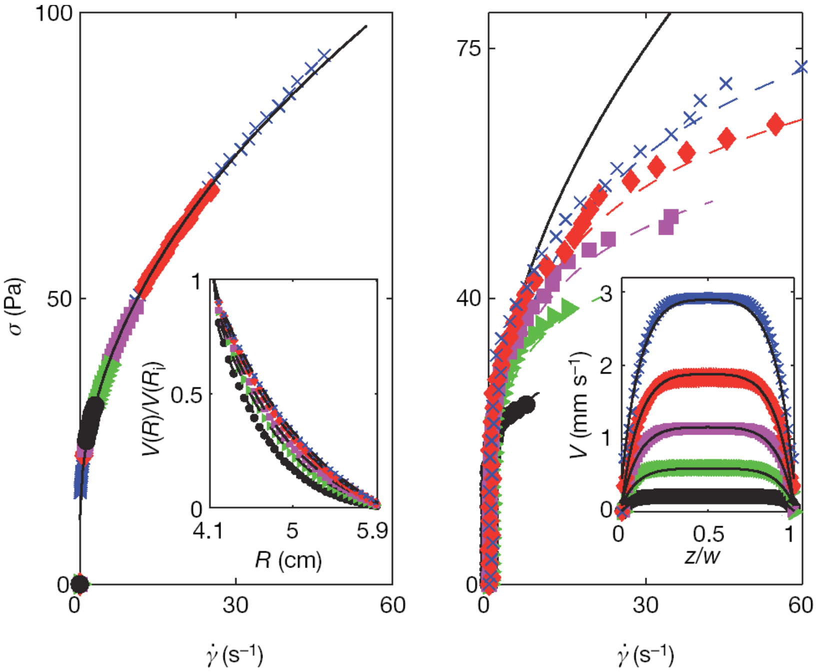


**Figure 16** : Flow of a concentrated emulsion (yield stress: 11.6 Pa, droplet size: 6.5 µm) in different geometries. (left) Flow curves extracted from the velocity profiles measured in a wide gap (2 cm) Couette cell (inset) at different rotation velocity of the inner cylinder. (right) Flow curves extracted from the velocity profiles (inset) measured in a channel of small width (250 µm). The flow cruves do not overlap for the channelized flows, likely because of a confinement effect, the ratio of the channel to the droplet size being too small. Data from Goyon [181].

*Surface tension effects*

As appear from the above description the characteristics of yield stress fluid flows along solid surfaces have generally been considered under the assumption that interfacial effects were negligible and viscous effects were dominant. For example the spreading of a yield stress fluid over an inclined solid plane is used as a technique for measuring the yield stress from the critical thickness at stoppage of a wide layer assuming that the gravity stress is simply balanced by the wall shear stress which, when the fluid just stops moving, is equal to the yield stress (see Section 5.1). The impact of wetting conditions on the flow of a yield stress fluid over a solid surface was studied only in very specific cases, i.e. drop impact [185-187], thin film flows [188], and these works already showed that original trends can be observed when both capillary effects and the complex solid-liquid nature of these materials play a role. On the contrary, for simple (Newtonian) liquids there is a well-developed background for describing thin layer spreading, coating, wetting, on solid surfaces, by taking into account surface tension, viscous and gravity effects [189].

One reason for the fact that viscous effects are generally assumed to be dominant in yield stress fluid flows is that we do not have clear information about the effective surface tension of such materials. For fluids made of elements in suspension in a simple liquid, such as foams, emulsions, suspensions, colloids, it was suggested that surface tension is simply equal to that of the interstitial liquid [13], since all the elements (bubbles, droplets, particles, etc) and in particular those situated along the fluid-air interface, are generally surrounded by liquid. Actually the above physical argument might fail for concentrated systems for which the liquid layer along the interface is very thin (such as in some emulsions or foams), or when the components of the liquid layer along the air-fluid interface are unknown (such as in some concentrated pastes).

More generally the role played by the elements which form a network of interactions at the origin of the yield stress may be critical during surface tension measurements. Indeed let us consider, in the vein of Israelachvili's reasoning [190], that we open a yield stress fluid sample into two parts so that we increase the area of the interface between the fluid and air. Some surface energy will as usual be required for this operation. However we will also need to provide some viscous energy to strongly deform the sample at least around the region of separation. It is known that the contact angle of simple liquids may be affected by non-negligible viscous effects but this effect disappears at vanishing velocity. Contrary to simple liquids, for a yield stress fluid the viscous energy does not tend to zero for a vanishing rate of deformation. Due to the fluid yield stress this energy tends to a finite value.

Measuring the surface tension of yield stress fluids thus constitutes a challenge. One work specifically focused on the measurement of surface tension of a gel from periodic laser irradiation of the surface [191] and found the value of the interstitial liquid. For simple liquids there exist various techniques: pendant drop, drop weight, maximum bubble pressure, Wilhelmy, du Nouy ring, etc. However, for the reasons abovementioned it is likely that they cannot readily be used for yield stress fluids without great care and further analysis of the data. Indeed they are based on the measure of residual stresses at rest or at sufficiently low velocities, which corresponds to situations for which there may exist finite residual stresses in the fluid. In their work dedicated to the determination of the elongational properties of yield stress fluids a couple of authors mentioned measurements of the surface tension of fluids exhibiting a sufficiently low yield stress [150, 152, 155] so that capillary effects could be dominant for small filament thickness.

A similar principle was used in a specific study of surface tension of yield stress fluids [153] from a series of tests consisting to withdraw a film from a bath of yield stress fluid by moving away a plate initially in contact with the fluid. It was shown that before a progressive breakage of the film, the force amplitude on the plate goes through a maximum which is independent of the initial depth of penetration and the timing for blade lifting, but increases with the material yield stress and the blade thickness. This critical force is shown to reflect both capillary and viscous effects, even at vanishing blade velocity. It was shown that the ratio of this force to the blade perimeter provides the surface tension of the yield stress fluid in the limit of low ($\ll 1$) Capillary number (ratio of yield stress times blade thickness to surface tension). It is worth noting that a study of the role of surface tension in the contact with soft elastic solids led to a similar result [192] : surface tension becomes dominant when the elastic modulus times the characteristic length is smaller than surface tension. Finally Carbopol gels appear [153] to have almost the same value of surface tension whatever their yield stress, but the value was found to be almost 10% smaller than that of pure water.

## 6. Conclusion

Due to their quite complex behavior including a solid and a liquid regimes the flows of yield stress fluids lead to a myriad of original features. The description and understanding of these flow characteristics are still rather difficult, precisely due to the coexistence of these two states in an a priori unknown way. With regards to the richness of this field there is finally a very limited amount of existing experimental data. Thus a wide field of research is still open, in particular concerning the internal flow characteristics.

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